

**BIOMEDICAL MATHEMATICS:**

**Promising Directions in Imaging,  
Therapy Planning, and Inverse Problems**

**BIOMEDICAL MATHEMATICS:**  
**Promising Directions in Imaging,  
Therapy Planning, and Inverse Problems**

**Yair Censor, Ming Jiang, Ge Wang  
Editors**

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Conference on Biomedical Mathematics**

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# Preface

This book brings together 27 state-of-the-art, refereed and subsequently revised, research and review papers, by leading experts and practitioners in mathematical methods in biomedical imaging, in intensity-modulated radiation therapy (IMRT) and in optimization and inverse problems. The emphasis is on trying to discover relations and connections between these fields that will enhance progress in each of them. As this volume shows, applicable mathematical work in these fields goes hand-in-hand with real-world applications and the mutual “technology transfers” between them leads to further progress.

The topics covered here include mathematical aspects and practical problems in current major and emerging technologies in diagnostic and therapeutic medicine and biology research. The contributed work signifies the interdisciplinary cooperation between mathematicians and scientists from medical physics, engineering, clinical medicine, and biology that leads to mathematically based better solutions of practical problems in biomedical imaging and IMRT.

The Huangguoshu National Park of China, Guizhou, China, under the leadership of Mr. Degang Yuan, President of the Huangguoshu Tourism Group Company, LTD, recognizing the importance of the field and the need for interaction between theoreticians and practitioners, and desiring to create a high-profile cultural activity at the Huangguoshu National Park, provided us with a special grant to organize the “Huangguoshu International Interdisciplinary Conference on Biomedical Mathematics—Promising Directions in Imaging, Therapy Planning, and Inverse Problems.” The Conference took place during November 3–9, 2008, in the breathtaking Huangguoshu National Park of China.

The Conference was conducted under the scientific auspices of the Mathematical Center of the Ministry of Education of China at Peking University (PKU) in Beijing, China; the Center for Computational Mathematics and Scientific Computation (CCMSC) at the University of Haifa, Haifa, Israel; the School of Biomedical Engineering & Sciences at the Virginia Polytechnic Institute and State University, Virginia, USA; and the Guizhou University in Guiyang, Guizhou, China.

Experts from around the world were invited and participated. They came from Australia, Canada, Germany, Israel, Italy, Japan, Korea, P.R. China, Spain, Sweden, Switzerland, the United Kingdom, and the United States of America. Most of the papers in this volume originated from the lectures presented at the Conference, while others were written in the wake of discussions held during the Conference.

## Preface

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It is our pleasure to heartily thank Mr. Degang Yuan and the team of the Huangguoshu Tourism Group, Peking University, and Virginia Tech for their devoted and efficient work throughout all phases of the preparations for the conference. In particular we thank Dr. Lingyin Zhao, Zongmin (Tracy) Mao, Haifeng (Mountain) Tong, and Gang (Leo) Xie, from the the Huangguoshu Tourism Group; Prof. Tie Zhou, Dr. Caifang Wang, Dr. Xin Jiang, and Yuanzheng Si, Yu Zhou, Shengkun Shi, from Peking University (PKU); and Deepak Bharkhada, from Wake Forest University, for their cooperation and extraordinary support in organizing and conducting the Conference. Many thanks are due to the referees whose work enhanced the final versions of the papers which appear here. Last but not least, we thank the participants of the Conference and the authors who contributed their work to this volume. We gratefully acknowledge the help of Ms. Betsey Phelps, Managing Editor, Medical Physics Publishing, Madison, WI, USA, for her and her team's work on the production of this volume.

We hope that researchers in applied mathematics, medical physics, biomedical imaging, and intensity-modulated radiation therapy will find this book a useful tool in their current research and development efforts.

Yair Censor, Ming Jiang and Ge Wang  
Haifa, Beijing, and Blacksburg, VA, respectively  
*January 31, 2010*



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Furthermore, medical images are used. 1 Fundamentals of Biomedical Image Processing. According to the Maxwell laws, accelerated (continuously changing its direction) charge induces electro-magnetic radiation: the electron would lose energy gradually spiraling inwards and collapsing into the nucleus. Within the Bohr model of the atom, there are certain shells where an electron can orbit its nucleus without releasing electromagnetic radiation. BIOMEDICAL MATHEMATICS: PROMISING DIRECTIONS IN IMAGING, THERAPY PLANNING, AND INVERSE PROBLEMS Y. Censor, M. Jiang, G. Wang, Editors © 2009 Medical Physics Publishing. All rights reserved. Statistical Iterative Reconstruction for X-Ray Computed Tomography. Bruno De Man. Statistical Iterative Reconstruction for X-Ray Computed Tomography. This represents a quest for the most likely image, given the measurements. Applying Bayes rule, using the monotonicity of the logarithm and dropping the constant term, this formulation is equivalent to the following optimization problem: ( ) reconstructed. image. Biomedical Mathematics: Promising Directions in Imaging, Therapy Planning, and Inverse Problems (ed 1). Madison: Medical Physics Pub Corp; 2009. pp. 1-18. 5. Smith A, Gillin M, Bues M, Zhu XR, Suzuki K, Mohan R, Woo S, Lee A, Komaki R, Cox J, Hiramoto K, Akiyama H, Ishida T, Sasaki T, Matsuda K. The M. D. Anderson proton therapy system. The patient images and segmentation may be shared under UCLA IRB approval of amendment on a case by case base. The 4D planning package is proprietary due to patent protection but compiled binary code may be available under special agreement. Authors' contributions.