AP Calculus: Friend or Foe?

By Susan Schwartz Wildstrom

It is often argued that there is a "rush to calculus" in the school curriculum. Students, teachers, and everyone else act as if learning calculus was the goal of school mathematics. Along the way, other things might be neglected, leaving students without a good foundation for future courses. Is AP Calculus a friend or a foe?

It is not the rush to calculus that has weakened the preparation our students have for doing advanced (never mind advanced, any) mathematics. Rather, this is the result of the weaker preparation of teachers who serve our elementary school students and the dilution of the curriculum of the other high school courses (algebra, geometry, precalculus). The 1989 standards emphasize relevance, use of real-world data, data collection, data analysis, and technology (replacing drill and practice). Pursuing these goals has taken a lot of class time away from the teaching of solid mathematical processes including algebraic operations and equation solving.

Let's start with elementary school. If students could complete elementary school with a mastery of the basic operations for whole numbers, decimals, and fractions, then their middle and high school experiences could be more productive.

I believe that children arrive in school as eager learners. We can encourage or discourage them by our attitudes and enthusiasms. The classroom teacher is a major force influencing how and what these students learn. If the teacher conveys the impression that math is hard, we should not be surprised that students internalize that attitude as well. (If the big person can't do something, how could little old me do it?)

I fear that many elementary school teachers have not been required to take enough mathematics to actually understand clearly how the mathematics works. They may have taken methods courses in which just one way to do a particular operation is taught. Later, when a clever student offers an unusual approach the most common teacher reaction is to say "No! That is not the way I taught it." While teachers should be wary of "dumb-luck" methods of getting an answer, they should also examine them carefully enough to assess if they are in fact valid. And, if the alternatives are valid, teachers should strive to understand how the alternate processes work and even try to incorporate them as a part of their future teaching.

Since the mid-1980s, I have argued for math specialists in every elementary school. No school would be without a reading specialist, why not a math specialist too? Some of the services that this person could provide include information about multiple methods for teaching a concept, remediation for students who are having difficulty mastering a concept, enrichment for students who are ready for extensions of grade-level concepts, teacher encouragement, support and in-service training.

Then there is our actual high school curriculum. Its organization seems to waste a lot of time teaching mathematics as just so many individual isolated processes, unrelated one to another. Students end up wasting a lot of their effort learning these concepts in isolation. When students actually *do* see the connections of processes to one another, they are able to learn more quickly and easily because their understanding of earlier concepts can be brought to the new ones.

Students (sometimes) learn, for example, to factor quadratic polynomials. They learn to solve quadratic equations using the quadratic formula. They learn to graph quadratic functions on their calculators. It is surprising how many of them do *not* realize that setting a factor to zero and solving produces the same value that the quadratic formula does and that that very value is the x-coordinate of the point where the graph is crossing the x-axis. Does this make sense?

In some textbooks, complex numbers are taught before simplification of radicals. Current algebra (1 and 2) textbooks no longer contain an important theorem that appeared in every algebra textbook when I started my career in 1969: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ if and only if $a, b \geq 0$. Are we surprised, then, that students who learn about complex numbers before simplifying radicals give us bizarre negative or complex valued answers for problems that should be straightforward? Several years ago, two colleagues who were teaching Algebra 2 in my school came and asked me why a student couldn't claim that $\sqrt{36} = -6$ by arguing as follows:

$$\sqrt{36} = \sqrt{-9} \cdot \sqrt{-4} = 3i \cdot 2i = -6.$$

My answer to them was to state the theorem about the values having to be non-negative, but when we looked at the book we were using as a textbook, it was nowhere to be found. I pulled out an older (1960s or earlier) text and, naturally, there it was. Alas, these days students getting teaching certification for grades 7–12 may have had a topology course (not part of my math major in 1969), but may not realize that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ if and only if $a, b \geq 0$.

All students now seem to have the technology — a graphing calculator is a standard requirement for most high school classes — but they seldom understand how to use it judiciously. They will turn it on to compute 9 x 5 yet they will return from the AP exam and ask me how they were supposed to find the point of intersection of $f(x) = 3x^2 + 2$ and $g(x) = \cos x$.

A friend and colleague of mine, who also taught calculus for many years, used to wonder why we taught calculus concepts in algebra 1 and 2. (Our school system has units on functions in the algebra course; they seem to leave the kids more bewildered about functions when they are done than when they started. These units include the notions of increasing, decreasing, and continuous functions; they also discuss maxima, minima, end-behavior, etc). If we were using the time we spend on those ideas to teach polynomial operations, factoring, solving equations, deeper understanding of these ideas and how they actually connect to one another, I believe the time would be better spent. In calculus we teach these concepts (increasing, decreasing, continuity, end behavior, maximum and minimum values) again, and I am not convinced that the students grasp them more quickly because of the earlier exposure. Maturity matters more.

I don't like having to stop in the middle of a calculus lesson to teach factorization of quadratic functions, why setting a factor equal to zero produces a root, and so many other topics that I always considered basic ideas of elementary algebra. I believe my calculus students would have a far easier time learning the difficult calculus notions if we didn't have the mid-topic distraction of having to review an algebraic step that is needed to complete a process.

I do believe that those students who survive calculus in our high school classes are likely to be well-served. If the course we offer in high school (which is likely to meet five days a week for forty-five minutes a class and for 36 weeks — I suspect that "block scheduling" is another fad that will eventually be cycled back out of schools) involves filling in the algebra and geometry gaps that are left by what I view as an inadequate curriculum, and if the course takes the time to explain not just how to get answers but what we are doing and why it works, then the students will arrive in math departments at universities ready for more advanced work.

The AP test does seem to do a fairly good job of detecting whether the test takers have some understanding of what calculus is about or not. Thus, students who score 4 and 5 seem to be demonstrating a readiness for college mathematics. I am not sure whether a 3 should still qualify a student to advance to a second semester course without additional proof of competency (perhaps the new MAPLE/MAA placement materials could serve that function), and I am certain that scores below 3 give evidence that a student should plan to retake calculus to build the depth of understanding that isn't yet there.

Should we be discouraging students from taking AP Calculus? Probably not. But, as I try to do, we should be looking at these people carefully early in the year, and if we see that they are too poorly prepared to succeed even with the extra algebraic support that we offer in the course, we should try to have them take a course that is more likely to help them build up the needed strength. In my school (and many others, I believe) we offer a non-AP, non-rigorous, "Calculus with Applications" class that helps students learn "how" to take derivatives and such while spending an even greater amount of time building the algebraic and trigonometric skills and understanding that is lacking. These students can enroll in Calculus 1 at their colleges with stronger basics and an introductory sense of the major concepts of calculus. They can, if they will, use this computational knowledge of limits, derivatives and integrals to help them take the time to learn more about the concepts that underlie these processes, and they too should be able to be successful in future mathematics courses.

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Friend or Foe. Thread starter Ryu. Start date Sep 28, 2005. < Previous | Next >. R. Ryu. Senior Member.Â (2) Do "them" and "us" generally mean "enemy" (or with the side of people who are against me) and "friend" (or with the side of people who are for me)? Click to expand Generally, yes. It's a way of dichotomizing. It's sometimes used to further demonstrate that two sides are opposing. Ryu said: (3) Which is more often used, "them or us" or "us or them"? friend or foe in uninformed play will adopt a conditional strategy: cooperate with opponents. whose attributes predict similarly cooperative behavior, and play foe against opponents whose. attributes do not.Â players, show little evidence of playing conditional strategies: Their foe rates do not differ ap-. preciably by opponent attributes in season 2. Within the context of the model, this implies the. foe-rate curve f0 for these groups is basically flat between the two equilibrium points in Figure 3. Two instructors, each teaching two sections of Calculus I, agreed to treat one of their sections as an experimental group and the other as a control group. Students in the experimental sections were issued an access code from the publisher of the textbook used in the course, to an online site that reflected the problems at the end of each of the text's chapters. Homework problems were assigned for almost all sections covered in the course, and students were allowed multiple attempts with feedback on problems submitted incorrectly. The assignments were graded and recorded by the computer.Â @inproceedings{Halcrow2012OnlineHI, title={Online Homework in Calculus I: Friend or Foe?}, author={Cheryl Halcrow and Gerri Dunnigan}, year={2012} }.