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THE RELATION BETWEEN THE RATE OF CHANGE OF MONEY WAGE AND UNEMPLOYMENT RATES IN THE FRAME OF THE MICROECONOMIC DYNAMICS[1]

Abstract: *The Phillips curve is a basic tool to understand relations between the growth rate of money wage and unemployment rates. Economic dynamics literature contains numerous precious analyses of the Phillips curve. Authors mostly automatically assume that the Phillips curve defines money wage growth rate, or inflation rate as a decreasing and convex function of unemployment rate, or output gap and as an increasing function of expected money wage growth, or expected inflation rate. Basic models of the economic dynamics are the original market equilibrium models. If the subject of dynamic analyses is labour market – market commodity is labour and its price is money wage – we can derive the Phillips curve. Books and lectures on economic dynamics could be enriched with this approach. Such analyses could help to understand why the Phillips curve is decreasing or why it should be augmented by expectations.*

Keywords: *Phillips Curve, cobweb models, dynamic adjustment models*

JEL: C 02, C 62, J 20

Introduction

The Phillips curve [14] represents the relationship between the rate of inflation and the unemployment rate. We can differ between the original and augmented Phillips curve. The original Phillips Curve had been considered to be a basic definition of the relation between inflation and unemployment for long time. The close fit between the estimated curve and the data caused that many economists, following the lead of Samuelson and Solow [17]. Edmund Phelps [12] and [13] and Milton Friedman [3] independently challenged theoretical underpinnings of the Phillips Curve. They argued that well-informed, rational employers and workers should pay attention only to real wages – the inflation-adjusted purchasing power of money wages. These

[1] This paper is supported by the Grant Agency of the Slovak Republic - VEGA, grant No. 1/0595/11 "Analysis of Business Cycles in the Economies of the Euro Area (with Regards to the Specifics of the Slovak Economy) Using Econometric and Optimization Methods"

long-run and short-run relations can be combined in a single "by-expectations-augmented" Phillips curve.

The expectations-augmented Phillips curve is a fundamental element of almost every macroeconomic forecasting model. Early new classical theories assumed that prices adjusted freely and that expectations were formed rationally – that is, without systematic error (see Lucas [9] and Sheffrin [18]). Some "New Keynesian" and some free-market economists as Taylor [21], Calvo [2] and Roberts [15] hold that, at best, there is only a weak tendency for an economy to return to non-accelerating inflation rate of unemployment. They specified a New-Keynesian Phillips Curve.Using real data, coefficients of this or similar specification were estimated in various papers as in Galí and Gertler [4] and Galí, Gertler and Lopez-Salido [5], Atkenson and Ohanian [1], Rudd and Whelan [16], Paloviita [11] and Miskin [10]. König [7] estimated a New-Keynesian Phillips curve using the Slovak data, Školuda [20] dealt with the hysteresis of the unemployment in Slovakia and other Visegrad economies.

When studying the economic dynamics we can discern two original dynamic microeconomic models (dynamic models of the market equilibrium). Model where price/quantity adjusts to excess demand (price/quantity adjustment model) and cobweb model. As we will show in the second section, the price-adjustment model of aggregate labour market can define the relation between money wage growth rate and unemployment rate. Expectations by supply (labour force) or demand (firms) can be formed in the cobweb model. As we will show in the third section, by means of combination of price-adjustment and cobweb models of aggregate labour market we can derive a by-expectations-augmented Phillips curve.

In our paper we will attempt to find some simple original dynamic models of the labour market equilibrium and derive a Phillips curve from them. For the sake of simplicity, we will consider linear models as the same or similar to them; those are analyzed in economic dynamics literature; we will refer to Gandolfo [6] or Shone [21]. We will assume that readers have some knowledge of economic dynamics that they can study in both books mentioned.

Our paper is segmented into three sections. Basic dynamic models are original dynamic models of the market equilibrium. We will briefly introduce them in the first section. We will deal with linear models of price or quantity adjustment and cobweb models. There are continuous and discrete versions of models. We will show both of them to demonstrate that they lead to different interpretations.

In the second section we will derive the original Phillips curve from the price-adjustment model. We will confront the Phillips curve with the theory. Our Phillips curve is, in contrary with the theory, concave, but this is caused only by linearity assumptions. In the end of the second section we will show a general case of the Phillips curve derivation that is consistent with the theory. Our analyses are, as in the first section, continuous and discrete. Since Phillips curve properties are characterized in equilibrium conditions, there are no significant differences between Phillips curves in continuous or discrete time.

Modern foundations of the Phillips curve in the frame of microeconomic dynamics are in the third section. We refer Phelps [12] who suggested extending the Phillips curve by vacation rate and expectations. Phelps showed that employment growth rate is described by inherent dynamic system, which is simultaneous to the one considered in the second section. Employment growth rate cannot be explained by quantity-adjustment model, if nominal money wage is explained by price-adjustment model of the aggregate labour market. Employment growth rate can be explained by another simultaneous system. According to this formulation it is possible that current vacancy rate does not equal to negative value of current unemployment rate, and so is more realistic. Since employment growth rate depends on population growth rate, in the Phillips curve analysis we should speak about the steady state of unemployment rate and steady state vacancy rate rather than about the equilibrium unemployment or vacancy rate.

Before we derive by-expectations-augmented Phillips curve we will analyze some models combined by price-adjustment and cobweb models. These combinations are described by the second order differential or difference equations. It is rather complicated to express their solution analytically; we will restrict our problem only to verify stability conditions of models. Finally, we will derive the by-expectations-augmented Phillips curve from combined models.

1 Original Dynamic Models of the Market Equilibrium

Everyone who studies economic dynamics can manage original dynamic models of the market equilibrium such as models of price or quantity excess demand adjustment or cobweb models. In this section we will briefly introduce these models.

The static linear model of the market equilibrium can be written as:

$$d(w) = a + bw \tag{1.1}$$

$$s(w) = a_1 + b_1 w \tag{1.2}$$

$$d(w) = s(w) \tag{1.3}$$

In the model d is a function of demand for a market commodity s is a function of supply for the commodity, w is its price a, b demand function coefficients and a_1, b_1 supply function parameters coefficients. It is clear that static equilibrium is given by equilibrium price $\overline{w} = (a - a_1)/(b_1 - b)$ and equilibrium quantity of a commodity $\overline{N} = (ab_1 - a_1 b)/(b_1 - b)$.

In general, markets are not in its equilibrium all the time. Therefore it is interesting to study the dynamics of the model; if the model is in its equilibrium, will it return to the equilibrium point and how long it will take?

The model (1.1)-(1.3) can be made dynamic if we assume that price/quantity adjusts to the demand excess (Walras/Marshall conception of price/quantity adjustment model), or that supply is affected by lagged price (cobweb model). All models can be analyzed in continuous and discrete time.

1.1 Continuous Models

Models of Price and Quantity Adjustment

According to this conception, we can extend a model by the mechanism of price adjustment in disequilibrium and we can rewrite the term (1.3) by (see Gandolfo [6] for more details):[2]

$$\dot{w} = \alpha\left(d - s\right) \tag{1.4}$$

where $\alpha > 0$ is an adjust coefficient, $(d - s)$ is an excess demand. By dotted symbols we denote the partial derivation of a variable subject to time. By substituting (1.1) and (1.2) to (1.4) we get first order differential equation

$$\dot{w} = \alpha\left(b - b_1\right)w + \alpha\left(a_1 - a\right) \tag{1.5}$$

Its solution can be written by price function of the time:

$$w\left(t\right) = \left(w_0 - \overline{w}\right)e^{\alpha\left(b-b_1\right)t} + \overline{w} \tag{1.6}$$

where w_0 is an initial price and $\overline{w} = (a - a_1)/(b_1 - b)$ is an equilibrium price. It is clear that the first term of (1.6) is a deviation of equilibrium price. This deviation vanish as $t \to \infty$ if $b < b_1$, so the stability condition is: $b < b_1$. We note that if slopes of demand and supply are normal ($b < 0$ and $b_1 > 0$), stability condition holds (see Gandolfo [6] for more details).

Model (1.1), (1.2) and (1.4) is known as Walras conception of price adjusting. There is also Marshall conception of quantity adjusting. According to this conception quantity adjusts to price excess demand function, which is defined as difference of price demand and price supply functions. Mathematically:

$$wd\left(N\right) = d^{-1} = -\frac{a}{b} + \frac{1}{b}N \tag{1.7}$$

$$ws\left(N\right) = s^{-1} = -\frac{a_1}{b_1} + \frac{1}{b_1}N \tag{1.8}$$

$$\dot{N} = \chi\left(wd - ws\right) \tag{1.9}$$

where N is quantity of the commodity, $\chi > 0$ is adjust coefficient, $(wd - ws)$ is price excess demand. By substituting (1.6) and (1.7) into (1.8) we get a first order differential equation and its solution can be written by a quantity function of the time:

[2] If we assume that it is linear.

$$N(t) = \left(N_0 - \bar{N}\right) e^{\chi \frac{(b_1 - b)}{b b_1} t} + \bar{N} \tag{1.10}$$

The Stability condition is $(b_1 - b)/bb_1 < 0$. We note that if slopes of demand and supply are normal, stability condition holds (see Gandolfo [6] for more details).

Original Cobweb Model

In the original cobweb model it is assumed that there is no adjustment mechanism of price (quantity), but supply (demand) is determined by lagged price. We can consider this model (linear case):

$$d(t + \mathrm{d}t) = a + bw(t + \mathrm{d}t) \tag{1.11}$$

$$s(t + \mathrm{d}t) = a_1 + b_1 w(t) \tag{1.12}$$

$$d(t) = s(t) \tag{1.13}$$

We can also write equilibrium identity (1.13) in the form: $d(t + \mathrm{d}t) = s(t + \mathrm{d}t)$. Demand, supply and price are differentiable functions of time. In the model the production of the market commodity takes some time (dt), but suppliers have to make their production decisions at the time of the beginning of the production (t). It is clear that we can $w(t + \mathrm{d}t)$ write as $w(t+\mathrm{d}t) = w + \dot{w}$. Then substituting (1.11) and (1.12) to (1.13) we get a first order differential equation:

$$\dot{w} = \frac{b_1 - b}{b} w + \frac{a_1 - a}{b} \tag{1.14}$$

Its solution can be written by the price function of the time:

$$w(t) = \left(w_0 - \bar{w}\right) e^{\frac{b_1 - b}{b} t} + \bar{w} \tag{1.15}$$

It is clear that the stability condition is $(b_1 - b)/b < 0$. We note that if slopes of demand and supply are normal, stability condition holds.

Cobweb Model with Adaptive Expectations

In real situation we should assume that producers behave more rationally and try to estimate future prices. They can expect that in time $t + \mathrm{d}t$ the price of the commodity will be $w^e(t)$. In model (1.11)-(1.13) then we rewrite supply function (1.12) by

$$s(t + \mathrm{d}t) = a_1 + b_1 w^e(t) \tag{1.16}$$

Let's assume that expectations are adaptive:

$$\dot{w}^e(t) = \gamma\left[w(t) - w^e(t)\right] \tag{1.17}$$

where $\gamma > 0$ is an adaptive expectation coefficient. The Cobweb model with adaptive expectations then consists from the demand function (1.11), the supply function (1.16), the equilibrium identity (1.13) and the adaptive expectations formula (1.17).

To solve this system we can write demand and supply functions as:

$$d + \dot{d} = a + b(w + \dot{w}) \tag{1.18}$$

$$s + \dot{s} = a_1 + b_1 w^e \tag{1.19}$$

From (1.18) and (1.19) it is clear that:

$$d = a + bw \tag{1.20}$$

$$s = a_1 + b_1\left(w^e - \dot{w}^e\right) \tag{1.21}$$

$$\dot{d} = b\dot{w} \tag{1.22}$$

$$\dot{s} = b_1 \dot{w}^e \tag{1.23}$$

Since in the model equilibrium (1.13) must in all time hold, also $d + \dot{d} = s + \dot{s}$ and $\dot{d} = \dot{s}$. Now let's express $\dot{w}^e$ from (1.23), w^e from (1.19) and substitute them to the adaptive expectations formula (1.17):

$$\frac{\dot{s}}{b_1} = \gamma w - \gamma\frac{s + \dot{s} - a_1}{b_1}$$

Since equilibrium holds, we have

$$\frac{\dot{d}}{b_1} = \gamma w - \gamma\frac{d + \dot{d} - a_1}{b_1}$$

Finally, if we substitute (1.18) and (1.22); we will get the first order differential equation:

$$\frac{b}{b_1}(1+\gamma)\dot{w} - \gamma\left(1 - \frac{b}{b_1}\right)w = \gamma\frac{a_1 - a}{b_1} \tag{1.24}$$

Its solution is price function of the time:

$$w(t) = \left(w_0 - \overline{w}\right)e^{\frac{\gamma(b_1 - b)}{b(1+\gamma)}t} + \overline{w} \tag{1.25}$$

It is clear that the stability condition is $\gamma(b_1 - b)/b(1 + \gamma) < 0$. We note that if slopes of demand and supply are normal, stability condition holds. In comparison with original cobweb model, the cobweb model augmented by expectations in general converges faster (if converges) since $\gamma/(1 + \gamma) < 1$ and so $\gamma(b_1 - b)/b(1 + \gamma) < (b_1 - b)/b$ (see Gandolfo [6] for more details).

1.2 Discrete Models

Analysing models in the continuous time is only one view on the dynamics of economic variables. According to this conception we should assume that economic system is continuous and is changing from second to second. Another view is to study economic systems using discrete models. By this approach we assume that economic variables change only once at a specific time. We will show that solutions of models from Part 1.1 can be in different in discrete time.

Models of Price and Quantity Adjustment

Walras linear model of price adjusting (1.1), (1.2) and (1.4) can be in discrete form written as:

$$d_t = a + bw_t \tag{1.26}$$

$$s_t = a_1 + b_1 w_t \tag{1.27}$$

$$\Delta w_t = w_{t+1} - w_t = \beta(d_t - s_t) \tag{1.28}$$

By substituting demand and supply functions (1.26) and (1.27) to price adjustment (1.28) we get the first order difference equation:

$$w_{t+1} = \left[1 + \beta\left(b - b_1\right)\right] w_t + \beta\left(a - a_1\right) \tag{1.29}$$

Its solution is price function of the time:

$$w\left(t\right) = \left(w_0 - \overline{w}\right)\left[1 + \beta\left(b - b_1\right)\right]^t + \overline{w} \tag{1.30}$$

The stability condition is given by:

$$\left|1 + \beta\left(b - b_1\right)\right| < 1 \tag{1.31}$$

From (1.31) it is clear that stability condition is b ä $(-2/\beta + b_1, b_1)$. This condition is stricter than one from the same continuous model ($b < b_1$). There is one more difference in comparison with continuous model. In discrete model the price can oscillate if the term in brackets is negative; $1 + \beta(b - b_1) < 0$. The Marshall model of quantity adjustment can be formulated and solved by a similar logic.

Original Cobweb Model

In discrete time original cobweb model is

$$d_{t+1} = a + bw_{t+1} \tag{1.32}$$

$$s_{t+1} = a_1 + b_1 w_t \tag{1.33}$$

$$d_t = s_t \tag{1.34}$$

The term of the equilibrium identity (1.34) we can also write in the form: $d_{t+1} = s_{t+1}$. By substituting demand and supply functions (1.32) and (1.33) to equilibrium identity (1.34) we get the first order difference equation:

$$w_{t+1} = \frac{b_1}{b} w_t + a_1 - a \tag{1.35}$$

Solution of (1.35) is:

$$w(t) = \left(w_0 - \bar{w}\right)\left(\frac{b_1}{b}\right)^t + \bar{w} \tag{1.36}$$

System is stable if absolute value of the demand slope is less than absolute value of the supply slope; ½b_1½ < ½b½. It is possible that in normal case the model is not stable. Price oscillates if $b_1/b < 0$. In normal case, since the demand slope is negative, supply slope is positive, price oscillates.

Cobweb Model with Adaptive Expectations

If producers expect price movement, we can rewrite the model in the form:

$$d_t = a + bw_t \tag{1.37}$$

$$s_t = a_1 + b_1 w^e_t \tag{1.38}$$

$$d_t = s_t \tag{1.39}$$

Adaptive expectations are expressed by the formula:

$$w_t^e - w_{t-1}^e = \delta\left(w_{t-1} - w_{t-1}^e\right);\ 0 < \delta < 1 \tag{1.40}$$

From (1.38) we can express w^e_t and w^e_{t-1}:

$$w_t^e = \frac{s_t - a_1}{b_1} \tag{1.41}$$

$$w_{t-1}^{e} = \frac{s_{t-1} - a_1}{b_1} \tag{1.42}$$

and substitute them along with equilibrium identity (1.39) and demand and supply functions, (1.38) and (1.37), to adaptive expectations formula (1.40) to get the first order difference equation:

$$w_{t+1} = \left[\left(\frac{b_1}{b} - 1 \right) \delta + 1 \right] w_t + \frac{(a_1 - a)\delta}{b} \tag{1.43}$$

Its solution is:

$$w(t) = (w_0 - \bar{w}) \left[\left(\frac{b_1}{b} - 1 \right) \delta + 1 \right]^t + \bar{w} \tag{1.44}$$

The stability condition is:

$$\left| \left(\frac{b_1}{b} - 1 \right) \delta + 1 \right| < 1 \tag{1.45}$$

From (1.45) we can get stability conditions in the form:

$$1 - \frac{2}{\delta} < \frac{b_1}{b} < 1$$

This condition is less strict as the original cobweb model stability condition ($-1 < b_1/b < 1$), since δ is less than one and more than zero and so $1 - 2/\delta < 1$. In normal case price oscillates.

We can see that in general continuous and discrete models differ. For the purpose of analysis it is important to select correct model to avoid incorrect interpretations. In general, adjustment models used to be continuous as price can adjust in continuous time. Production period, on the contrary, is defined by a specific time and so cobweb models are usually discrete.

2 Original Phillips Curve Derivation

In this section we will show, how we can derive the original Phillips Curve from adjusting price model of labour market if we use Lipsey's [8] logic – demand excess is negative unemployment.

2.1 Continuous Time

We will consider a continuous Walras linear model of price adjustment (1.1), (1.2) and (1.4). Let the analyzed market be an aggregate labour market; commodity on market is working time, its price is money wage[3]. On the supply side of the

[3] The aggregate assumptions may in some cases lead to incorrect interpretations. Phelps [12] shows that our models are not these cases.

labour market is labour force and on the demand side are firms. We will use the same denomination as we have used till now. From the theory of the labour market we can assume that demand and supply slopes are normal; $b < 0$ and $b_1 > 0$. From the first section we already know solution of the model and we know that the motion of money wage is monotonic and it converges to equilibrium.

How can we derive the Phillips Curve from this model? The Phillips curve is a relation between the rate of money wage change and unemployment rate. We can express the first one as $\dot{w}/w$, the second we can express, as we will show, by excess demand.

Let's modify the model by rewriting price-adjusting formula (1.4) by:

$$\frac{\dot{w}}{w} = \varepsilon\left[d(w) - s(w) + \bar{U}\right];\ \varepsilon > 0 \tag{2.1}$$

In comparison with the price adjusting formula (1.5), in (2.1) we made two modifications. One is that we assume that the equilibrium identity (1.3) does not hold in equilibrium. In general, according to the theory, there is some amount $\bar{U}$ of labour force that does not work if the economy is in its potential. Then equilibrium is expressed by the formula $d(w) = s(w) - \bar{U}$ instead of $d(w) = s(w)$.

The second modification is that we rewrote $\dot{w}$ in (1.4) by $\dot{w}$/w. The reason of this modification is to make calculations simpler. We can show that this slight modification does not dramatically change properties of the model. The model (1.1), (1.2) and (2.1) can be expressed by the first order nonlinear differential equation:

$$\dot{w} = \alpha(b - b_1)w^2 + \alpha\left(a_1 - a + \bar{U}\right)w \tag{2.2}$$

The first order differential equation (2.2) is nonlinear. Equilibrium is different only because of existence of unemployed labour force $\bar{U}$, $\bar{w} = (a - a_1 + \bar{U})/(b_1 - b)$ instead of $\bar{w} = (a - a_1)/(b_1 - b)$ and equilibrium demand $d(\bar{w}) = (ab_1 - a_1b + b\bar{U})/(b_1 - b)$ and supply $s(\bar{w}) = (ab_1 - a_1b + b_1\bar{U})/(b_1 - b)$ instead of $\bar{N} = (ab_1 - a_1b)/(b_1 - b)$. Using phase diagram of (2.2) we can see that the stability condition does not change (see Shone [21] for more details).

Now, let's derive the Phillips curve. Let's define unemployment as $s(w) - d(w)$ and unemployment rate as:

$$u(w) = \frac{s(w) - d(w)}{s(w)} \tag{2.3}$$

Here we make one important assumption. We assume that if there exists unemployment in the economy the labour demand is fulfilled; and on the other hand, if there are vacancies in the economy all labour force is employed. According to this assumption the vacancy rate is negative value of the unemployment rate; $v = -u$. This assumption certainly is not realistic and was one of the subjects of Phelps's [12] critique that we will show in the third section.

In our model unemployment rate is:

$$u(w) = \frac{(a_1 - a) + (b_1 - b)w}{a_1 + b_1 w} \tag{2.4}$$

For constructing the Phillips curve we need to find the relation between the change of money wage rate $\dot{w}/w$ and the unemployment rate u. If we substitute (2.3) to (2.1) we can get:

$$\frac{\dot{w}}{w} = \varepsilon\bar{U} - \varepsilon u s(w) \tag{2.5}$$

In our model:

$$\frac{\dot{w}}{w} = \varepsilon\bar{U} - \varepsilon u(a_1 + b_1 w) \tag{2.6}$$

Express from (2.4) money wage as a function of the unemployment rate:

$$w(u) = \frac{a_1 - a - a_1 u}{b - b_1 + b_1 u} \tag{2.7}$$

If we substitute (2.7) to (2.6) we will get the Phillips curve for our model:

$$\frac{\dot{w}}{w} = P(u) = \varepsilon\bar{U} - \varepsilon u\left(a_1 + b_1 \frac{a_1 - a - a_1 u}{b - b_1 + b_1 u}\right)$$

or:

$$\frac{\dot{w}}{w} = P(u) = \varepsilon\bar{U} + \varepsilon\frac{(ab_1 - a_1 b)u}{b - b_1(1-u)} \tag{2.8}$$

It is easy to verify that function $P(u)$ is decreasing. Let's see what unemployment rate is. In the equilibrium $\dot{w} = P(u) = 0$, so if the right side of (2.8) is zero, the relation:

$$\bar{u} = \frac{(b_1 - b)\bar{U}}{ab_1 - a_1 b + b_1\bar{U}} \tag{2.9}$$

must hold. We already know that equilibrium supply is $s(\bar{w}) = (ab_1 - a_1 b + b_1\bar{U})/(b_1 - b)$ and so we can (2.9) write to get the definition of the natural unemployment rate as:

$$\bar{u} = \frac{\bar{U}}{s(\bar{w})} \tag{2.10}$$

The Phillips curve intersects x-axis, in agreement with the theory, in the natural unemployment rate point.

2.2 Discrete Time

Let's consider the discrete model (1.26)-(1.28). We can rewrite (1.28) by:

$$\frac{\Delta w_t}{w_t} = \varepsilon\left[d\left(w_t\right) - s\left(w_t\right) + \bar{U}\right] \tag{2.11}$$

We will define unemployment rate as:

$$u\left(w_t\right) = \frac{s\left(w_t\right) - d\left(w_t\right)}{s\left(w_t\right)} \tag{2.12}$$

If we substitute demand and supply functions (1.26) and (1.27) to (2.13) and express money wage we get:

$$w\left(u_t\right) = \frac{a_1 - a - a_1 u_t}{b - b_1 + b_1 u_t} \tag{2.13}$$

and finally substitute (2.14) and (2.13) to (2.12) to get discrete form of the Phillips curve:

$$\frac{\Delta w_t}{w_t} = P\left(u_t\right) = \varepsilon\bar{U} + \varepsilon\frac{\left(ab_1 - a_1 b\right)u_t}{b - b_1\left(1 - u_t\right)} \tag{2.14}$$

We can see that there are no differences between continuous and discrete conceptions of the Phillips curve. When we analyse the Phillips curve, we study its equilibrium. Equilibrium is same in both continuous and discrete time.

2.3 General Case

Till now we have for assumed the sake of simplicity that our relations are linear. In this case the resultant Phillips curve is decreasing and intersects x-axis in the natural unemployment rate point. According to the theory, the Phillips curve should be convex, but it is easy to verify that Phillips curve that comes from linear model is concave. This is caused by the linearity assumption, which we made for simplicity.

In the general case we can consider a labour demand curve $d(w)$ which, according to the theory, is decreasing and concave, a labour supply curve $s(w)$ is increasing and convex and finally an adjustment price function $\dot{w}/w = \varepsilon(x + \bar{U})$ (where $x = d(w) - s(w)$ is excess demand). About the adjustment price function we only know that it is increasing. From demand and supply functions we can define unemployment rate as a function of the money wage (2.3). From (2.3) it is possible to express money wage as a function of the unemployment rate $w(u) = u^{-1}(w)$. If we substitute $w(u)$ to the adjustment price function we will get the Phillips curve in the general form:

$$\frac{\dot{w}}{w} = P\left(u\right) = \varepsilon\left\{d\left[w\left(u\right)\right] - s\left[w\left(u\right)\right] + \bar{U}\right\} \tag{2.15}$$

The Phillips curve (2.11) should be decreasing and convex and should intersect x-axis in the natural unemployment rate point.

3 Augmented Phillips Curve

In the second section we derived the original Phillips curve, which has been considered by economists as the basic relation between money wage change and unemployment rates for a long time. Phelps [12] showed that the logic behind the Phillips curve derived at section 2 is not correct.

3.1 Vacancy Rate

First we assumed that vacancy rate is a negative unemployment rate, and so there are no job vacancies if there is positive no zero unemployment rate and there is no unemployment if there is positive non zero job vacancies. Phelps showed that there is relationship between money wage change rate and both the unemployment and the vacancy rate. If we use his denomination it means that

$$\frac{\dot{w}}{w} = m(u,v) \tag{3.1}$$

where v is the vacancy rate; $m(u,v)$ is twice differentiable function, $m_u < 0$, $m_v > 0$, $m_{uu} \geq 0$, $m_{vv} \geq 0$ and $m_{uv} \leq 0$.

Phelps showed that for vacancy rate we could substitute the absolute time rate of increase of the aggregate number of persons employed. To show this we need to study employment dynamics. Perhaps one possibility is to form some variety of Marshall Conception of quantity (employment) adjustment.

By this conception if there is equilibrium in the economy, the unemployment rate supply of labour and demand for labour are in the equilibrium too. However, as Phelps showed, this need not be true. The change in the number of employees consists of the number of persons hired less the departures of employed persons and quitting of employees to join the unemployed in search of new jobs. It is clear (see Phelps [12] for more details) that such defined change of the number of employers will be the function of the unemployment and vacancy, mathematically:

$$\dot{N} = N(U,V) \tag{3.2}$$

where V is job vacancies. Dividing by (3.2) by labour supply we get function for the rate of the change of employers:

$$z = \frac{\dot{N}}{s(w)} = z(u,v);\ z_u > 0, z_v > 0 \tag{3.3}$$

Equation (3.3) is first order differential equation describing an autonomous dynamic system. Let's assume constant nonnegative population growth $n = \dot{L}/L$, where L is population size. If current economy has unemployment rate in amount u, steady state employee growth rate will be:[4]

$$\bar{z} = (1-u)n \tag{3.4}$$

Now if we express vacancy rate as a function $\psi(u, z)$ and substitute it to (3.1) we will get an augmented Phillips curve:

$$\frac{\dot{w}}{w} = m\left[u, \psi(u,z)\right] = f(u,z) \tag{3.5}$$

In the Phillips curve (3.5) we assume: $f_u < 0, f_{uu} > 0, f_v > 0, f_{vv} > 0$ and $f_{uv} < 0$. For constant z the Phillips curve is decreasing and convex, but intersects x-axis in the point which depends on z. "The variables u and z cannot go their own way for long since high (low) z implies a falling (rising) u. There is, therefore, some interest in the steady state." Tu nie je uvedený zdroj The steady state of unemployment rate $\bar{u}$ and the steady state of employment growth rate are given by:

$$\bar{z} = (1-\bar{u})n \tag{3.6}$$

By substituting (3.6) to (3.5) we define a steady state Phillips curve as $f[\bar{u}, (1-\bar{u})n]$, which is negatively sloped and steeper than the constant-z Phillips curves:

$$\frac{\partial f\left[\bar{u},(1-\bar{u})n\right]}{\partial \bar{u}} = f_u - f_z < f_u < 0$$

3.2 Expectations

In our model from the second section we assumed that decisions of subjects of the labour market were not affected by time lags. Actually, wage contracts are made for long term, and we should assume that both firms and labour force expect the future movement of money wage. The Phillips curve than comes from a combination of price adjusting and cobweb models. For the sake of simplicity, we will again assume that the vacancy rate is negative value of the unemployment rate.

Firstly, we will examine the dynamics of combined models and then we will derive the Phillips curve. We will consider both continuous and discrete models. We note that actually the cobweb part of the model should be discrete and price-adjustment part of the model should be continuous. Results of models, which are clearly continuous or clearly discrete, may be misleading. The system, which is a combination of continuous and discrete parts, is characteristic of mixed differential and difference equation. Solving this type of equations is rather complicated.

[4] Here it is correct to speak about steady state rather than equilibrium, since in the steady state employment grows in dependence of population growth n, as one can see in term (3.4).

Then we will derive a Phillips curve. We will study the Phillips curve only in equilibrium and so it does not matter whether model is continuous or not.

The Combination Price-Adjustment and Cobweb Models

Continuous model

Let's consider a model in the form:

$$d(t + \mathrm{d}t) = a + bw^e(t) \tag{3.7}$$

$$s(t + \mathrm{d}t) = a_1 + b_1 w(t) \tag{3.8}$$

$$\dot{w}^e(t) = \gamma\left[w(t) - w^e(t)\right];\ \gamma > 0 \tag{3.9}$$

$$\dot{w} = \alpha\left[d(t) - s(t)\right];\ \alpha > 0 \tag{3.10}$$

Firms have some expectations about future money wage movement, which determine demand (3.7). These expectations are formed adaptive (3.9). From (3.8) is clear that in the model (3.7)-(3.10) we have made an extreme Keynesian assumption that labour force does not expect any change of the money wage.

Let's express a differential equation. We can (3.7) write as:

$$d + \dot{d} = a + bw^e \tag{3.11}$$

and so:

$$d = a + b\left(w^e - \dot{w}^e\right)$$

and

$$\dot{d} = b\dot{w}^e \tag{3.12}$$

About expected expectations we know from (3.11):

$$w^e = \frac{d + \dot{d} - a}{b} \tag{3.13}$$

and from (3.12):

$$\dot{w}^e = \frac{\dot{d}}{b} \tag{3.14}$$

Next, express demand from price adjustment formula (3.10):

$$\dot{w}+\ddot{w}=\alpha\left[\left(d+\dot{d}\right)-\left(s+\dot{s}\right)\right] \tag{3.15}$$

and

$$\ddot{w}=\alpha\left(\dot{d}-\dot{s}\right) \tag{3.16}$$

Then we have expressions for the demand:

$$d+\dot{d}=\frac{\dot{w}+\ddot{w}}{\alpha}+s+\dot{s} \tag{3.17}$$

and

$$\dot{d}=\frac{\ddot{w}}{\alpha}+\dot{s} \tag{3.18}$$

Finally from (3.8) we can get:

$$\dot{s}=b_1\dot{w} \tag{3.19}$$

Now substitute expectations (3.13) and (3.14) to adaptive expectations formula (3.9):

$$\frac{\dot{d}}{b}=\gamma\left(w-\frac{d+\dot{d}-a}{b}\right)$$

Then substitute demand (3.17) and (3.18)

$$\frac{\ddot{w}}{\alpha b}+\frac{\dot{s}}{b}=\gamma w-\gamma\frac{\dot{w}+\ddot{w}}{\alpha b}-\gamma\frac{s+\dot{s}}{b}+\gamma\frac{a}{b}$$

And finally substitute supply (3.8) and (3.19):

$$\frac{\ddot{w}}{\alpha b}+\frac{b_1\dot{w}}{b}=\gamma w-\gamma\frac{\dot{w}+\ddot{w}}{\alpha b}-\gamma\frac{a_1+b_1 w}{b}+\gamma\frac{a}{b} \tag{3.20}$$

If we make some modifications in (3.20) we can get a second order differential equation in the form:

$$\frac{1}{\alpha b}\left(1+\gamma\right)\ddot{w}+\left(\frac{b_1}{b}+\frac{\gamma}{\alpha b}\right)\dot{w}+\gamma\left(\frac{b_1}{b}-1\right)w=\gamma\frac{a-a_1}{b} \tag{3.21}$$

Let's find equilibrium solution of the second order differential equation (3.21). In equilibrium:

$$\gamma\left(\frac{b_1}{b}-1\right)\overline{w}=\gamma\frac{a-a_1}{b}$$

After some modification:

$$\overline{w} = \frac{a - a_1}{b_1 - b} \tag{3.22}$$

Next, the characteristic equation of the second order differential equation homogenous to (3.21) is (see Gandolfo [6] for more details):

$$\frac{1}{\alpha b}(1+\gamma)\lambda^2 + \left(\frac{b_1}{b} + \frac{\gamma}{\alpha b}\right)\lambda + \gamma\left(\frac{b_1}{b} - 1\right) = 0 \tag{3.23}$$

To find the general solution of the homogenous equation to (3.21) we should analytically express roots of the characteristic equation (3.23). As we know (see Gandolfo [6]), roots of the characteristic equation may be real distinct, real multiple or complex conjugate, and so the price movement may be monotonic or oscillated. We still can examine the stability of the model without solving characteristic equation (3.23). All the three coefficients in the characteristic equation are negative. The stability condition then is fulfilled (see Gandolfo [6] for more details) and our system is stable.

Discrete Model

The discrete variation of the model (3.7)-(3.10) we can write in the form:

$$d_t = a + bw^{e}_{t} \tag{3.24}$$

$$s_t = a_1 + b_1 w_{t-1} \tag{3.25}$$

$$w^{e}_{t} - w^{e}_{t-1} = \delta(w_{t-1} - w^{e}_{t-1}) \tag{3.26}$$

$$w_t - w_{t-1} = \beta(d_t - s_t) \tag{3.27}$$

Now express expectations from the demand function (3.24):

$$w_t^e = \frac{d_t - a}{b} \tag{3.28}$$

and

$$w_{t-1}^e = \frac{d_{t-1} - a}{b} \tag{3.29}$$

Then express demand from the price adjustment formula (3.27):

$$d_t = \frac{w_t - w_{t-1}}{\beta} + s_t \tag{3.30}$$

and

$$d_{t-1} = \frac{w_{t-1} - w_{t-2}}{\beta} + s_{t-1} \tag{3.31}$$

And substitute (3.28) and (3.29) for expectations, then (3.30) and (3.31) for demand to the adaptive expectations formula (3.26), to get, after elementary modifications, twice difference equation:

$$w_t + \left(\delta - 2 + \beta b_1 - b\beta\delta\right) w_{t-1} + \left(1 - \delta + b_1\beta\delta - \beta b_1\right) w_{t-2} = \beta\delta\left(a - a_1\right) \tag{3.32}$$

Let's find equilibrium solution:

$$\overline{w} + \delta\overline{w} - 2\overline{w} + \beta b_1\overline{w} - b\beta\delta\overline{w} + \overline{w} - \delta\overline{w} + b_1\beta\delta\overline{w} - \beta b_1\overline{w} = \beta\delta\left(a - a_1\right)$$

$$\overline{w} = \frac{a - a_1}{b_1 - b}$$

The characteristic equation of the second order differential equation homogenous to (3.32) is (see Gandolfo [6] for more details):

$$\lambda^2 + \left(\delta - 2 + \beta b_1 - b\beta\delta\right)\lambda + 1 - \delta + b_1\beta\delta - \beta b_1 = 0 \tag{3.33}$$

To find the general solution to the homogenous equation to (3.32) we should analytically express roots of the characteristic equation (3.33). As we know (see Gandolfo [6]), price movement may be monotonic or oscillated. We can still examine the stability of the model without solving characteristic equation (3.33).

We can write stability conditions as (see Gandolfo [6] for more details):

$$1 + \delta - 2 + \beta b_1 - b\beta\delta + 1 - \delta + b_1\beta\delta - \beta b_1 = 0 \tag{3.34}$$

$$1 - \left(\delta - 2 + \beta b_1 - b\beta\delta\right) > 0 \tag{3.35}$$

$$1 - \left(\delta - 2 + \beta b_1 - b\beta\delta\right) + 1 - \delta + b_1\beta\delta - \beta b_1 > 0 \tag{3.36}$$

Since coefficients δ and β are positive, the first stability condition (3.34) can be written as b - $b_1 < 0$. Since demand and supply slopes are normal, this condition is fulfilled.

The second stability condition (3.35) can be written as $3 - \delta - \beta b_1 + b\beta\delta > 0$, which means:

$$b_1 < \frac{3 - \delta\left(1 - b\beta\right)}{\beta} \tag{3.37}$$

The third stability condition (3.36) can be written as $4 - 2\delta - 2\beta b_1 - \beta\delta(b - b_1) > 0$ and so:

$$b_1 < \frac{4-\delta\left(2-b\beta\right)}{\beta\left(2-\delta\right)} \tag{3.38}$$

We can again conclude that in discrete model stability conditions are stricter. By contrast to original models, combination of price adjustment and cobweb models are described by second order equations, and so there is possibility of the oscillated movement of price and quantity in continuous as well as in discrete time.

In models (3.7)-(3.10) and (3.24)-(3.27) we made the assumption that labour force does not predict the future movement of the price – supply is function of current price. This assumption is extreme Keynesian. One can modify models by assumptions of adaptive (Keynesian) or perfect (neoclassic) expectations. Solving a model with adaptive expectations both in supply and demand functions is more complicated. By assumption of perfect expectations of the labour force, supply is simply a function of the future price ($w(t + \mathrm{d}t)$ in continuous case and w_{t+1} in discrete case). It is also possible to assume that firms have perfect assumption and so demand is function of the future price. Furthermore, there are plenty other possibilities, how to form expectations and adaptive expectations.

By-expectations-augmented Phillips Curve

From the models above we can derive the Phillips curve. Without loss of generality we can assume that labour force has perfect expectations of future price movements. We made this assumption to make the Phillips curve derivation simpler. Assumptions about adaptive or no expectations make this derivation complicated, but lead to similar derivations of the Phillips curve. Let's use continuous model, for steady state analysis it does not matter whether the model is continuous or discrete or the combination of both. Consider then model:

$$d(t+\mathrm{d}t) = a + bw^e(t) \tag{3.39}$$

$$s(t) = a_1 + b_1 w(t) \tag{3.40}$$

$$\dot{w}^e\left(t\right) = \gamma\left[w\left(t\right) - w^e\left(t\right)\right];\ \gamma > 0 \tag{3.41}$$

$$\frac{\dot{w}}{w} = \varepsilon\left[d\left(w, \dot{w}^e\right) - s\left(w\right) + \bar{U}\right];\ \varepsilon > 0 \tag{3.42}$$

The model (3.39)-(3.42) is similar to the model (3.7)-(3.10); w^e are adaptively formed expectations of firms. Excess demand (3.42) is for simplicity same as one we considered in the original Phillips curve derivation in the second section, (2.1).

By combination of the demand function (3.39) and the adaptive expectations formula (3.41) we can get the demand function as function of the money wage and function of the change of its expectations by firms:

$$d\left(w,\dot{w}^e\right)=a+bw-b\left(1+\frac{1}{\gamma}\right)\dot{w}^e \tag{3.43}$$

Since b is negative demand is positive function of the change of money wage expectations by firms. Further analysis is similar as the one in the second chapter. Let's define unemployment function as:

$$u\left(w,\dot{w}^e\right)=\frac{s\left(w\right)-d\left(w,\dot{w}^e\right)}{s\left(w\right)} \tag{3.44}$$

In our model:

$$u\left(w,\dot{w}^e\right)=\frac{\left(a_1-a\right)+\left(b_1-b\right)w+b\dfrac{1+\gamma}{\gamma}\dot{w}^e}{a_1+b_1w} \tag{3.45}$$

If we substitute unemployment rate (3.44) with excess demand (3.42) we will get (2.5). In our model we can write it by (2.6) and express from unemployment rate (3.44) money wage as a function of the unemployment rate and the change of its expectations by firms:

$$w\left(u,\dot{w}^e\right)=\frac{a_1-a-a_1u+b\dfrac{1+\gamma}{\gamma}\dot{w}^e}{b-b_1+b_1u} \tag{3.46}$$

Finally, if we substitute (3.46) with (2.6) we will get the Phillips curve for our model:

$$\frac{\dot{w}}{w}=\pi\left(u,\dot{w}^e\right)=\varepsilon\bar{U}+\varepsilon\frac{\left(ab_1-a_1b\right)u-b\dfrac{1+\gamma}{\gamma}\dot{w}^e}{b-b_1\left(1-u\right)} \tag{3.47}$$

The Phillips curve (3.47) $\pi\left(u,\dot{w}^e\right)$ differs from the original Phillips curve $P(u)$ (2.8) only by expectations, it is decreasing in u and increasing in $\dot{w}^e$. In equilibrium, $\dot{w}=\dot{w}^e=\pi\left(u,0\right)=0$, unemployment rate is natural and can be written by (2.9) and (2.10), since $\pi\left(u,0\right)=P(u)=0$.

In the literature the augmented Phillips curve defines the rate of money wage change as the function of unemployment rate and expectations of the money wage rate in the form:

$$\frac{\dot{w}}{w}=P\left(u\right)+\frac{\dot{w}^e}{w} \tag{3.48}$$

or in more general case:

$$\frac{\dot{w}}{w}=g\left(u,\frac{\dot{w}^e}{w}\right) \tag{3.49}$$

The Phillips curve is decreasing in u and is increasing in $\dot{w}^e/w$. Finally, if we consider employment growth dynamics (see above) we can derive Phelps's augmented Phillips curve write in the form:

$$\frac{\dot{w}}{w} = h\left(u, z, \frac{\dot{w}^e}{w}\right) \tag{3.50}$$

Conclusion

In our paper we showed how we can use original dynamic microeconomic models, known from the economic dynamics, to explain some basic properties of the Phillips curve. We can derive the original Phillips curve from the original price-adjustment model of the labour market equilibrium. From a combination of the cobweb model augmented by expectations and the original price-adjustment model we can derive a by-expectation-augmented Phillips curve.

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