INTRODUCTION TO FOURIER ANALYSIS

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SUMMARY

Comprehensive, user friendly, and pedagogically structured

A fast, easy way to learn about the electrical engineer's most important mathematical tool

Based on a groundbreaking one-semester course originated by Professor Norman Morrison at the University of Cape Town, this book serves equally well as a course text and a self-study guide for professionals. Offering only relevant mathematics, it covers all the core principles of electrical engineering contained in Fourier analysis, including the time and frequency domains; the representation of waveforms in terms of complex exponentials and sinusoids; complex exponentials and sinusoids as the eigenfunctions of linear systems; convolution; impulse response and the frequency transfer function; magnitude and phase spectra; and modulation and demodulation.

* Covers Fourier analysis exclusively for electrical engineering students and professionals
* Offers a complete FFT system (contained on the enclosed disks long for IBM compatibles, the other for Macintosh)
* Includes dozens of examples drawn from electrical engineering
* Packed with exercises, samples, and end-of-chapter problem sets

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