The Quantum Negative Energy Problem Revisited

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1 Introduction

The aim of this note is to revisit the problem of the negative energy levels in the early Dirac electron theory. It is a problem that was not resolved satisfactorily in the context of Dirac’s relativistic quantum theory, or in its extension to quantum electrodynamics, according to Dirac’s own view. It will be my contention that this problem is automatically resolved in any continuous field theory of microscopic matter, as in the author’s demonstration of quantum mechanics as a linear, asymptotic limit of a continuous, nonlinear field theory of the inertia of matter, rooted in the theory of general relativity, [1].

2 Review of the Quantum Electron Problem

In Dirac’s generalization of Schrodinger’s nonrelativistic wave mechanics, to a spinor form in special relativity, he was left with the problem of the existence of negative energy levels $E_-$, accompanying the positive energy levels $E_+$ of the electron. It was a problem not encountered in classical field theory or in nonrelativistic wave mechanics. According to Dirac [2],

“In the quantum theory, since in general a perturbation will cause transitions from states with $E$ positive to states with $E$ negative, such transitions would appear experimentally as the electron (whose charge) suddenly changes from $-e$ to $+e$, a phenomenon which has not been observed”.

In time, it was recognized that $+e$ (the positron) in the Dirac theory, is an elementary particle that is independent of the elementary particle $-e$ (electron). But the problem remained that in the context of the quantum
theory, the electron with positive energy, $E_+ = c \sqrt{m^2 c^2 + p^2}$ must, with a definite non-zero probability, drop to a negative energy state with energy $E_- = -c \sqrt{m^2 c^2 + p^2}$. The minimum gap between the positive and the negative energies of the electron, $(E_+ - E_-)_{\text{min}} = 2mc^2$, corresponding to zero electron momentum, $p = 0$, in each domain. In dropping to lower energy states, the electron would then continually lose energy until reaching the state at negative infinity - implying that matter could not be stable!

Dirac then postulated that the problem would be overcome by assuming at the outset that a separate electron already occupies each of the negative energy levels. Then, according to the Pauli exclusion principle, a positive energy electron could not drop into any of the negative energy states since other electrons occupy them all.

This model then left open the possibility that a negative energy electron near the top of this domain (i.e. near $E = -mc^2$) could be excited to an unoccupied positive energy state, thereby leaving a ‘hole’ in the negative energy state. Other negative energy electrons at lower energy levels could then fill the latter. An external electric field would then reveal what is equivalent to ‘hole conduction’ – the conduction of positive electrons in this domain.

Thus, Dirac’s resolution of the negative energy problem required that whenever one postulates the existence of single positive energy electron, it must be accompanied by an infinite number of negative energy electrons. The situation was not satisfactory to Dirac as a permanent resolution to the problem!

Of course, one may postulate ad hoc that the minimum energy of the electron is $+mc^2$, and that the ground state (zero) energy would be the vacuum state. But this could not be done in the context of the Dirac theory, because it would reduce the completeness of the latter in its applications to physical problems. For example, the derivation of the (empirically correct) Klein-Nishina formula for Compton (electron-photon) scattering [3], requires the complete set of negative and positive energy states.

Dirac next considered the extension of quantum mechanics to quantum electrodynamics to resolve the problem. In the latter theory, the ‘perfect vacuum state’ is postulated to be at zero energy, where there are no electrons, positrons or photons. But he showed that this was also not acceptable because the ‘perfect vacuum’ is not stationary[4]. At the end of Dirac’s book, he says [5]:

“It would seem that we have followed as far as possible the path of logical development of the idea of quantum mechanics as they are at present understood. The difficulties being of a profound character can be removed
only by some drastic change in the foundations of the theory, probably as
drastic as the passage from Bohr’s orbit theory to the present quantum
mechanics”.

3 A Resolution – Back to a Continuous Field Theory

It has been my contention that the “change in the foundations of the
theory” that Dirac alludes to would be a returning to the continuous field
theory of matter, rooted in the theory of general relativity [1]. For in this
view, the energy of a “particle of matter” (which, in the continuous field
theory is a mode of a single continuum) is defined according to the
Lagrangian formalism, according to Noether’s theorem, as:[1]

\[ E = \int \sum_{\alpha=1}^{n} \left[ \partial L / \partial (\partial_{\alpha} \Lambda^{(1)}) \right] \partial_{\alpha} \Lambda^{(1)} - L \, d^3x \]  

(1)

where \( L \) is the Lagrangian density for the continuum material system, \( \partial_{\alpha} \) is
the time derivative and \( \Lambda^{(1)}_{\alpha} \) are the \( \alpha \) components of the \( n \)-dependent
fields of the system of matter. The latter, in the Dirac electron theory, would
be his bispinor solutions \( \psi \), their hermitian conjugate solutions multiplied by
the Dirac matrix, \( \psi \gamma^0 \), and their respective derivatives. The dependent
fields, \( \Lambda_{\alpha} \), are generally all of the solutions of the field laws of nature. One
arrives at the definition (1) for the conserved ene
gry from the assumption
that the corresponding Lagrangian density \( L \) is invariant with respect to
arbitrary, continuous variations of the time parameter \( \delta t \).

Thus, energy is defined in the field theory in terms of continuous change
only. Since one value of energy \( E \) projects to another continuously connected
value of energy, \( E + \delta E \), for, say, an electron matter field \( \psi \), one may not
proceed from a positive value energy to a negative value energy as this
would be a discontinuous change. This is the reason, as Dirac asserts[2], that
one may automatically reject the negative energy values in a classical field
theory.

At this juncture in the history of quantum mechanics, in 1928, the route
taken was to quantum electrodynamics. But, as Dirac explained, this did not
work! In addition to the problem that the vacuum state in quantum
electrodynamics (\( \text{qed} \)) is not stationary, another of the critical reasons for its
failure was that extension to \( \text{qed} \) automatically generated infinities in the
formalism, thereby destroying the self-consistency of the theory. It was for
this reason that Dirac referred to \( \text{qed} \) as an “ugly theory” [6]. I have
previously also discussed some of these difficulties with \( \text{qed} \) [7].
I have demonstrated this route in resolving the problem of quantum mechanics in my research program[1]. In this view, quantum mechanics is a linear (asymptotic) approximation, at low energy, for a generally nonlinear field theory of the inertia of matter in general relativity. These results indeed confirm Dirac’s assertion [8]:

“Some day, a new quantum mechanics, a relativistic one, will be discovered, in which we will not have these infinities occurring at all. It might very well be that the new quantum mechanics will have determinism in the way that Einstein wanted ... I think it is quite likely, or at any rate quite possible, that in the long run Einstein will turn out to be correct.”

References


Note added in Proof:

Later research that addresses Dirac’s problem of a non-stationary vacuum is on the theory of field operator algebra, as discussed in G. G. Emch,

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These negative energy densities lead to several problems such as the failure of the classical energy conditions, the production of closed timelike curves and faster than light travel, violations of the second law of thermodynamics, and the possible production of naked singularities. Although quantum field theory introduces negative energies, it also provides constraints in the form of quantum inequalities (QIs). These uncertainty principle-type relations limit the magnitude and duration of any negative energy. In quantum field theory, there exist states in which the expectation value of the energy density for a quantized field is negative. Quantum theory, however, allows negative energy. According to quantum physics, it is possible to borrow energy from a vacuum at a certain location, like money from a bank, says Daniel Grumiller. For a long time, we did not know about the maximum amount of this kind of energy credit and about possible interest rates that have to be paid. Various assumptions about this interest (known in the literature as Quantum Interest) have been published, but no comprehensive result has been agreed upon. The so-called Quantum Null Energy Condition (QNEC), which was proven in 2017, prescribes certain